Reflection with a Twist: The Helical Mirror

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Over the last few years, the helical mirror or "spinner" has become a popular decoration for gardens and elsewhere. Even casual observation reveals intriguing optical properties, so the spinner is a good teaching tool. To facilitate student exploration, we suggest some questions and provide brief explanations.

How Do Image and Object Orientations Compare?

At first glance, the image in a vertically hung spinner appears to be rotated by 90° relative to the object. However, a comparison of the images in the spinner and the plane mirror behind it in Fig. 1(a) shows that the image in the spinner is actually inverted along a 45° diagonal. This inversion is particularly clear when the spinner's axis is held at 45°, as in Fig. 1(b). In this orientation, the horizontal cross section is convex and the vertical one is concave.[1] The image is inverted along the concave cross section (for an object outside its focal length) but not along the convex cross section.[2]

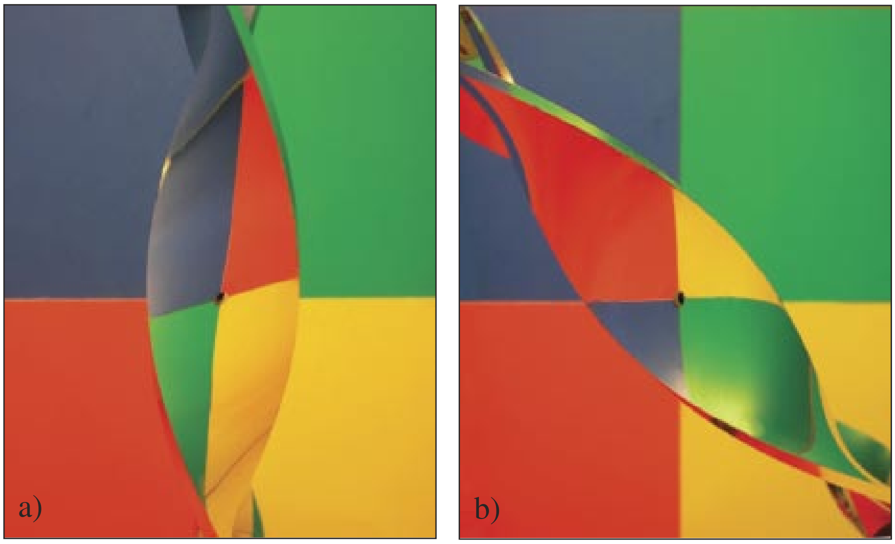

Fig. 1. Reflections in the spinner and a plane mirror. (a) Vertical spinner, as it is normally displayed and (b) Diagonal spinner, with the convex (concave) axis horizontal (vertical).

What Is the Image Distance?

For the spinner, the image distance is ambiguous. For simplicity, consider a point object outside the concave cross section's focal length. Rays from the object that reflect off the concave cross section converge on a point in front of the surface, but rays that reflect off the convex cross section diverge from a point behind the surface, so the image is astigmatic. Since these two image points are only a few centimeters apart for a typical spinner,[3] a single eye focused at an intermediate depth perceives a fairly clear image. When viewed with two eyes, the angle between the directions in which the eyes gaze, or "angle of convergence," gives another depth cue. If the observer's head is oriented so that the interocular axis (the line connecting the eyes) is parallel to the concave (convex) cross section, the two gazes converge at the corresponding image point in front of (behind) the mirror. At other orientations, the gazes do not converge, which can be dizzying.[4]

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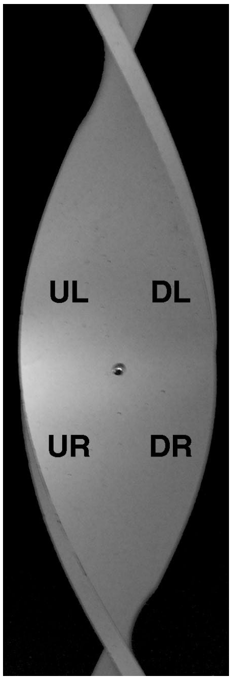

Fig. 2. The quadrants of a spinner segment face different directions, which are labeled U, D, R, and L to indicate up, down, right, and left, respectively.

How and Why Is Image distorted?

While the image reflected in a spinner suffers surprisingly little transverse distortion, there is noticeable longitudinal distortion. For example, an observer's reflection is neither abnormally broad nor lean, but the nose is a bit long. For a typical spinner, the concave and convex cross sections have nearly equal curvatures, so their focal lengths are approximately the same size but opposite sign. Magnification in terms of the magnitude of the focal length f and object distance d_{obj} is

$$M = \frac{f}{f \pm d_{obj}},$$

where the positive (negative) sign is for the convex (concave) cross section. For distant objects ($d_{obj} >> f$), the magnitudes of both magnifications are approximately f/d_{obj}, so there is little distortion in the height or width. [There is a sign difference, which indicates that one axis is inverted and the other is not, as shown in Fig. 1(b).] The presence of d_{obj} in the denominator means that more distant objects are less magnified, which also occurs for other magnifying mirrors and lenses.[5] This makes the image of a more distant object appear even farther away, so the image appears elongated.

How Does an Image Move as the Mirror Spins?

As a left-handed[6] spinner rotates clockwise (as viewed from above), the image travels downward and to the left. An observer's eye is a convenient object for exploring this since its image is seen at a point where the mirror's surface is perpendicular to the observer's gaze. Suppose the image is initially at the center of a segment, marked by the dot in Fig. 2. Points further below the dot face more rightward, and points further to the left of the center face more upward. When the spinner is rotated clockwise (i.e., all points on the mirror face more leftward), a new point is angled toward the observer. Relative to the dot, this new point must face slightly rightward so it is below the dot. Since the observer must now gaze down, the new point also must face slightly upward so it is left of the dot. Therefore, the observer's image travels downward and to the left across the spinner's surface as it rotates clockwise.

Whether it is in the garden or the classroom, we hope that the preceding questions will encourage others to take a closer look at the spinner. An interesting extension for students would be comparing the spinner to the cylindrical mirror and the double mirror (two perpendicular plane mirrors). We recommend the referenced papers for those seeking more detailed discussions of the optical phenomena touched on here.

References

1. The concave and convex axes are not exactly perpendicular due to a gradual change in the number of twists per length.
2. Alan J. DeWeerd and S. Eric Hill, "Reflections on handedness," *Phys. Teach.* **42**, 275–279 (May 2004).
3. The image points for the two cross sections are bounded by the convex cross section's focal point and the concave cross section's center of curvature.
4. Alan J. DeWeerd and S. Eric Hill, "The dizzying depths of the cylindrical mirror," *Phys. Teach.* **43**, 90–92 (Feb. 2005).
5. Héctor Rabal, Nelly Cap, and Marcelo Trivi, "Longitudinal magnification drawing mistake," *Phys. Teach.* **42**, 31–33 (Jan. 2004).
6. That is one that twists in the opposite way as a common right-handed screw.

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