Comment on "Anamorphic images," by J. L. Hunt, B. G. Nickel, and Christian Gigault [Am. J. Phys. 68 (3), 232–237 (2000)]

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Hunt, Nickel, and Gigault[1] concluded their paper on anamorphic images with a brief discussion of the ambiguous location of an image in a convex cylindrical mirror. They also made some observations of the effects of this ambiguity on viewing the image. In a recent paper[2] we explored the same ambiguity for a concave cylindrical mirror and the resulting peculiarities associated with viewing. We draw on that work to clarify the nature of the convex cylindrical mirror's image and to explain some observations of anamorphic images.

For simplicity, consider a point object reflected from a convex cylindrical mirror with its axis vertical. As illustrated in Fig. 1, the rays reflected from a linear, vertical cross section project to point a, while those reflected from a curved, horizontal cross section project to point b. To understand the nature of the reflection from the entire mirror, other vertical and horizontal cross sections must be considered. Rays reflected from all horizontal cross sections project to points that form a vertical line image passing through point a. On the other hand, rays reflected from all vertical cross sections project to points that form a curved line image[3] in the same horizontal plane as the point object. Because a single point is projected to two perpendicular lines, the image is astigmatic. The separation of the two image lines increases as the dis-

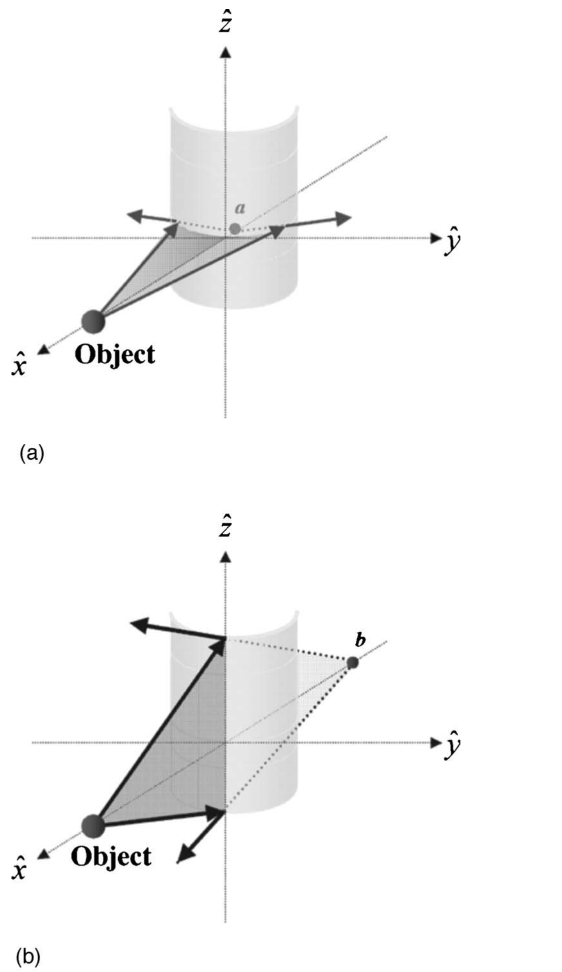

Fig. 1. Rays reflected by (a) a horizontal or (b) a vertical cross section of the cylindrical mirror project to different points.

tance between the object and mirror increases. A consequence of this worsening astigmatism is that parts of an anamorphic image viewed higher in the mirror appear less sharp.

The two line images are helpful for understanding how an astigmatic image is viewed. As shown in Fig. 2, a ray reaching an observer's eye must project through both line images.[4] When viewing the image with two eyes, the angle at which their gazes converge provides a depth cue. If two eyes are oriented so that the interocular axis is horizontal (vertical), then the two gazes converge on the vertical (horizontal) line image.[5] Therefore, the image of the point object either appears to stand on the vertical line or to lie on the horizontal one. This effect accounts for the observation in Ref. 1 that the anamorphic image appeared to stand up when viewed with the head held upright, but appeared to recline in the x-y plane when viewed with the head tilted.

Hunt *et al.*[1] also reported that the effect due to the head's orientation persisted when one eye was closed. This observation is intriguing because the above explanation only applies to binocular viewing. We explored two approaches to monocular observations.

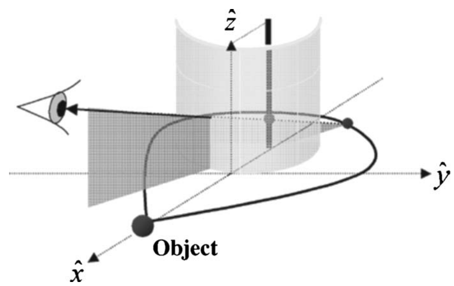

Fig. 2. A ray entering an observer's eye projects through the two line images.

When starting with one eye open, we did not notice any difference as the head's orientation was changed. When viewing with one eye, the amount of accommodation necessary to focus gives a depth cue. For an astigmatic image, the eye tends to balance the clarity along the two axes and focus at a point between the two line images. Regardless of the head's orientation, this intermediate focus would give the anamorphic image an inclination between the standing and reclining inclinations observed with two eyes, which is what we observed. While making these observations we were very careful to keep the eye at the same location. There is a natural tendency to lower the eye while rotating it by tilting the head sideways, which reduces the viewing angle and decreases the apparent inclination of the anamorphic image.

We also started with both eyes open and observed what happened when one eye was closed without changing the head's orientation. In both orientations, we noticed the sensation of the one eye changing its focal length upon closure of the other. However, closing one eye did not always create the strong perception that the inclination changed. When the interocular axis was vertical, the image no longer appeared to lie horizontally after either eye was closed. In this case, the viewing angles are not the same for the two eyes, but the difference can be minimized by viewing from a large distance. On the other hand, with the interocular axis horizontal, the inclination of the image did not appear markedly changed when either eye was closed. An explanation of this final observation may require going beyond physical cues to psychological ones.

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[1] J. L. Hunt, B. G. Nickel, and Christian Gigault, "Anamorphic images," Am. J. Phys. **68**(3), 232–237 (2000).
[2] Alan J. DeWeerd and S. Eric Hill, "The dizzying depths of the cylindrical mirror," Phys. Teach. **43**(2), 90–92 (2005).
[3] The curve is the inner loop of a limaçon. See the Appendix to Ref. 2 which is available at the EPAPS homepage as document E-PHTEAH-43-012502: http://www.aip.org/pubservs/epaps.html
[4] Figure 13 and Eq. (12) of Ref. 1 relate to the x-z plane of our Fig. 2. Their s' is the distance from the z axis to our vertical image line and their z is the height at which the gaze crosses that image line.
[5] If the eyes are at other orientations, then the gazes do not converge.