Conservation of Angular Momentum with Slightly Modified Commercial Apparatuses

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A spinning ice-skater who speeds up when she pulls in her arms is a common textbook illustration of how changing a rotating object's moment of inertia affects its angular speed. An analogous classroom demonstration involves sitting on a rotating stool while moving hand-held weights in and out. Given that students have difficulties understanding conservation of angular momentum when the moment of inertia changes,[1] we wanted to add a quantitative experiment that closely resembled the typical textbook example and in-class demonstration. There have been a few such experiments published: an apparatus that moves masses radially relative to a person rotating on a stool,[2] a rotating track along which model cars move radially inward or outward,[3] and a modified, centripetal-force apparatus with masses that slide outward.[4] The first two were more elaborate than we wanted, but the third one seemed to be a suitable experiment for an introductory lab. However, as described, that experiment required substantial modifications to a relatively expensive apparatus.[5] We have found two ways to perform the experiment by making simple modifications to less expensive commercial apparatuses.

In both cases, the important components are a vertical shaft that rotates without much friction, a way to collect angular position data with a computer interface, and a beam perpendicular to the shaft and along which two masses can slide. Additionally, two secure stops are essential to keep those masses from flying off the ends of the beam and potentially harming nearby students or computers. Initially, the masses are held near the central shaft by a thread connecting them. The system is set spinning by hand, then the thread is burned allowing the masses to slide outward to the stops. Since the moment of inertia increases, the angular speed decreases. The published apparatus uses springs to push the sliding masses outward, but we've found that those are unnecessary if there isn't too much friction between the masses and the beam.

One approach is to use a rotary motion sensor[6] and an accessory set,[7] as is shown in Figure 1. The sensor measures the angular position of its shaft. The accessory set is designed for other rotational experiments but can, with only minor modifications, be used for the desired experiment as well. The set includes a pulley for experimentally determining the moment of inertia, a round beam that's attached through a hole in its center to the rotary motion sensor, and two cylindrical masses that can slide along it. The beam has an additional hole near one end, and we drilled a symmetric hole (5/32-inch diameter) near its other end. To keep the masses from sliding off, 1/4-inch hitch pins[8] were inserted in these two holes. The original thumbscrews that were used to hold the cylindrical masses in place were replaced with shorter (3/8-inch long) bolts and washers which are used for attaching thread to connect them, but don't keep them from sliding.

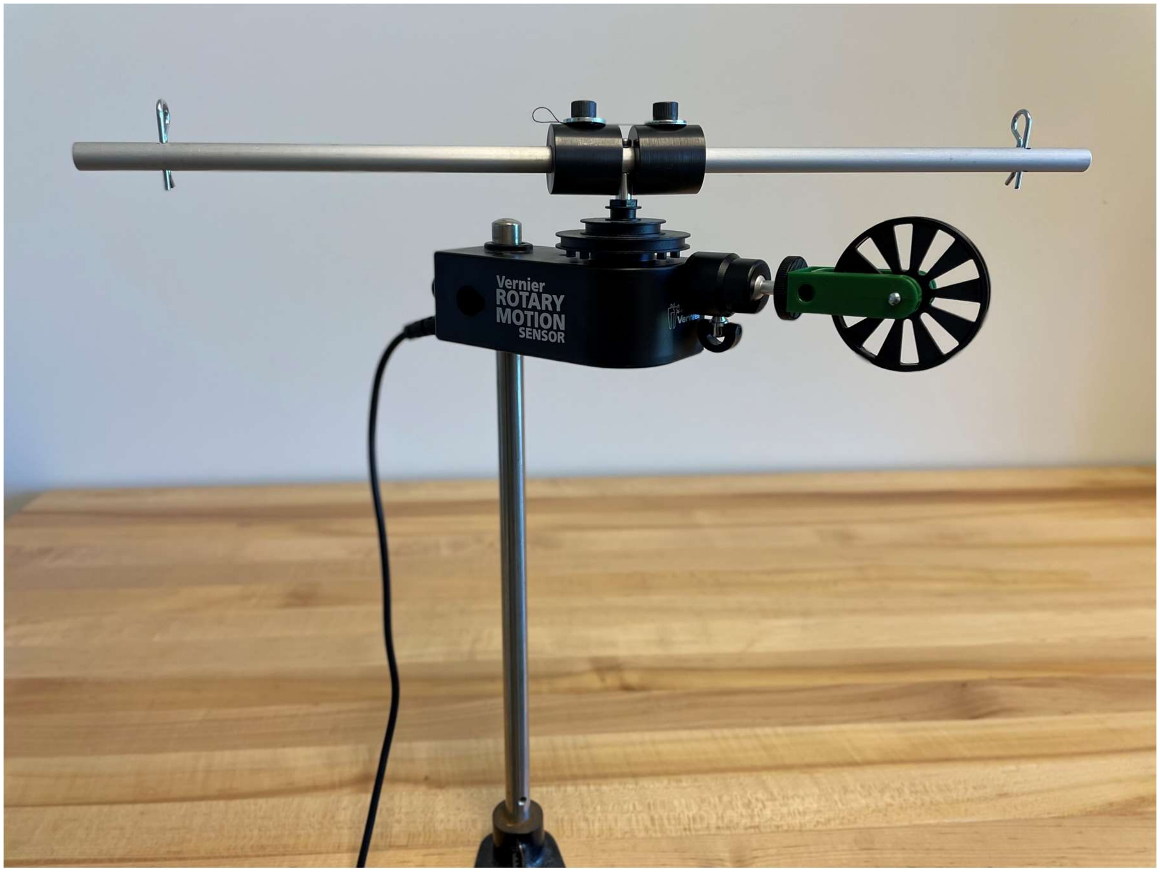


Fig. 1. The setup using a Vernier rotary motion sensor and accessory set. The beam came with a hole at the center and one near an end. A symmetric hole was added near the other end, and hitch pins were used as stops. The thumb screws in the cylindrical mass were replace with shorter ones and washers to hold the connecting thread without locking the masses in place.

Another option is to use the low-friction rotating platform, pulley, and photogate from Pasco,[9] which is shown in Figure 2. The rotating platform is a beam with a T-slot on top. We used 100-g masses from the Mass and Hanger Set[10] with either low-friction mass holders from the Wireless Centripetal Force Accessory[11] or 3D-printed sliders that we designed.[12] Threaded holes at each end of our sliders take 1/4-inch long 8-32 bolts and washers for clamping down a thread connecting them. The rotating platform comes with two short thumb screws and square nuts that go in the T-slot to act as stops. However, we found that those don't always hold when the sliding masses collide with them because they rely on friction. Instead, we drilled two 1/4-inch diameter holes below the T-slot 24 inches from the center and used longer (1-inch) thumb screws which extended through the holes. Pasco suggests using the unmodified apparatus for an experiment where a mass is pulled inward, but their experiment is more susceptible to error because students must be careful not to apply torque while pulling the mass in and friction matters more since the change in moment of inertia occurs more slowly. Also, the experiment that we've described is more dramatic, which students enjoy.

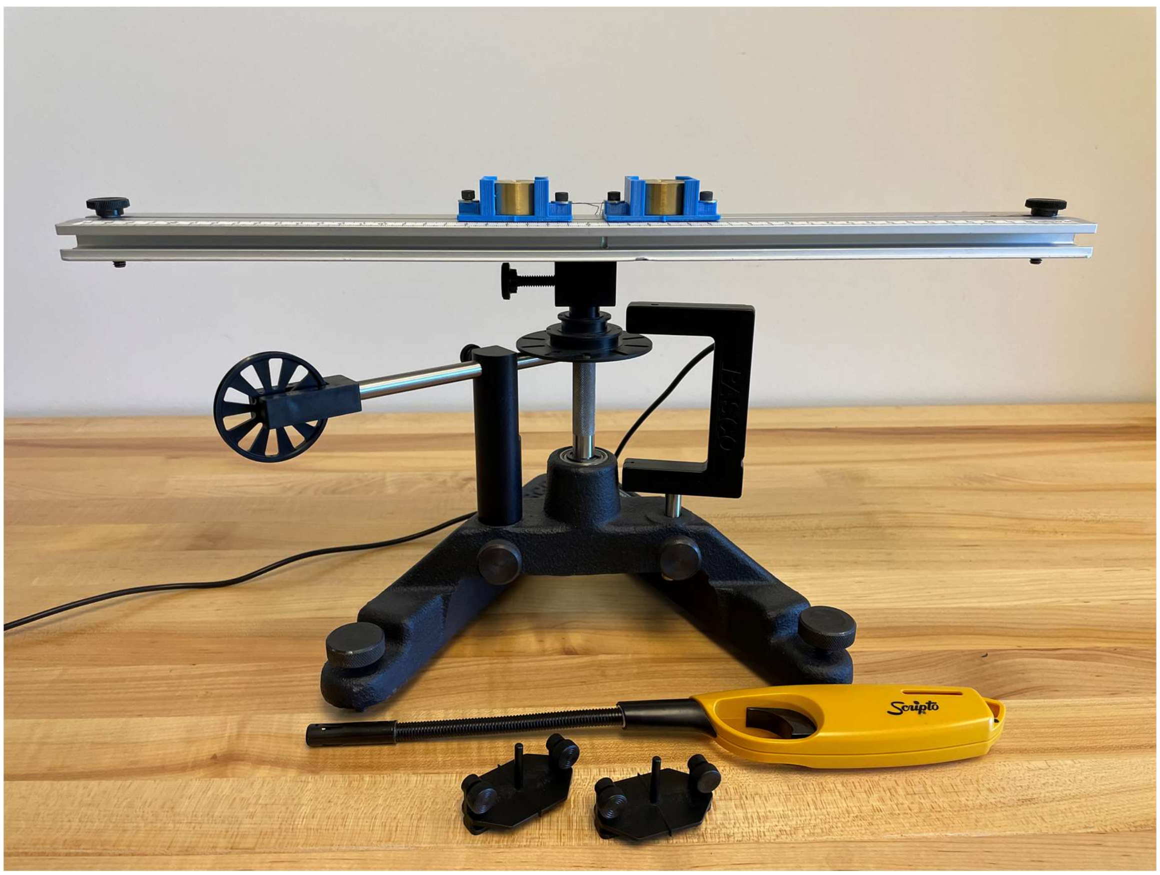


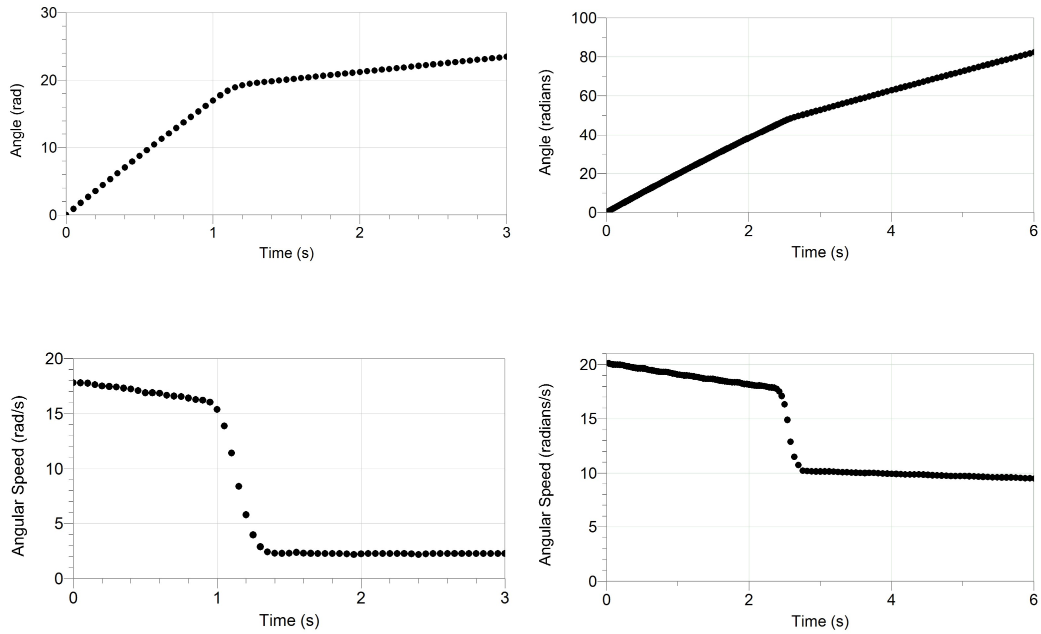
Fig. 2. The setup using Pasco's rotating platform. The two end stops can be seen extending through the track which ensures they don't fly off when struck by the mass holders. The blue, 3D-printed mass holders are shown on the track and connected by a thread. Pasco's black mass holders and the lighter used to burn the thread are on the tabletop.

It may also be possible to perform our experiment with Vernier's centripetal force apparatus[13] by purchasing a second sliding carriage [14], but we have not tested that. Anyone who tries this should be sure that the "end pieces" on the beam are securely mounted and won't fly off when struck by the sliding carriages.

The earlier paper on this experiment[4] covers the theory and analysis very well, so we don't repeat that here. However, there is one difference in how we approach the analysis. They had students take data with the sliding masses in the initial and final positions to determine the initial and final moments of inertia. Instead, we have our students take data to find the moment of inertia of the apparatus without the sliding masses or stops, I_{beam}, then calculate the total moment of inertia using

$$I \approx I_{beam} + 2m_{slider}r_{slider}^2 + 2m_{stop}r_{stop}^2 , \tag{1}$$

where r_{slide} and r_{stop} are the radial distances to the centers of the sliding masses and stops, respectively. This reinforces the idea of how moments of inertia combine for a compound system. In their analysis, students minimize the effect of friction by comparing angular speeds immediately before and after the thread is burnt and the masses slide to the stops. Sample data taken with the two modified apparatuses are shown in Figure 3. The angle vs. time graphs show that the changes in the angular speed occurs very quickly. Since the software (Vernier's Logger Pro) uses multiple data points to calculate the angular speed, the changes in the angular speed vs. time graphs are less sharp. Our students routinely observe conservation of angular momentum to within a few percent.



(a) **(b)**

Fig. 3. Sample angle and angular speed vs. time data for (a) the rotary motion sensor and (b) the rotating platform.

This conservation of angular momentum experiment has been a popular staple in both our algebra-based and calculus-based introductory laboratories, providing hands-on experience with determining a compound system's moment of inertia and the effect that changing the moment of inertia has on angular speed. The modifications suggested don't prevent the apparatuses from being used as the manufacturers intended, but they may void the warranties on them. We hope that our explanation of the modifications will help others add the experiment to their lab courses.

Endnotes

1. K.K. Mashood and Vijay A. Singh, "Rotational kinematics of a rigid body about a fixed axis: development and analysis of an inventory," *Eur. J. Phys.* **36**, 1-20 (2015).
2. Robert B. Prigo and Melissa Reading, "Quantitative angular momentum experiment on the rotating chair," *Am. J. Phys.* **45** (7), 636-637 (1977).

3. Wathiq Abdul-Razzaq and Leonardo Golubović, "Demonstrating the conservation of angular momentum using model cars moving along a rotating rod," *Phys. Ed.* **48** (1), 42-50 (2013).
4. Robert Carr, Harold Cohen, and Terry Ragsdale, "Demonstrating Angular Momentum Conservation," *Phys. Teach.* **48** (3), 169-171 (1999).
5. Beck Centripetal Force Apparatus, Daedalon® ($1,384.85 from Science First®).
6. We used the Rotary Motion Sensor (RMV-BTD, $189) by Vernier, but their Go Direct® Rotary Motion Sensor (GDX-RMS, $189) is similar. There are also three versions available from Pasco: Rotary Motion Sensor (CI-6538, $220), PASPORT Rotary Motion Sensor (PS-2120A, $185), and Wireless Rotary Motion Sensor (PS-3220, $198).
7. We used Vernier's Rotational Motion Accessory Kit (K-RMV, $120), but Pasco's Rotational Inertia Accessory (ME-3420, $125) is almost identical.
8. These were purchased at a hardware store.
9. Rotating Platform (ME-8951, $450), Super Pulley with Mounting Rod (ME-9499, $32) and Photogate Head (ME-9498A, $52).
10. Mass and Hanger Set (ME-8979, $99) or 100-g Replacement Mass Set (ME-8985, $400), which is a set of 6.
11. Wireless Centripetal Force Accessory (ME-8094, $36), two are required.
12. An STL file is available at https://www.thingiverse.com/thing:5426789.
13. Centripetal Force Apparatus (CFA, $599).
14. Sliding Carriage (SC-CFA, $14).